The SGfast Mex Function

Code and Documentation Written by:

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Overview

The *SGfast* function is a replacement function for the spike generator *SGmodel* function in Carney's AN Model code. It is an inhomogeneous Poisson process with a fixed absolute refractory period and a two-exponential relative refractory period, just like the aforementioned *SGmodel* function and the spike-generator implementations in Zhang et al. (2001) and Carney (1993). *SGfast*, however, is much faster and makes much more efficient use of the pseudo-random number generator. More specifically, (1) *SGmodel* uses a Bernoulli approximation to the Poisson process in each time bin, while *SGfast* uses the faster time-transformation method. This has the added benefit that it requires significantly fewer pseudo-random numbers (one per spike versus one per time bin for *SGmodel*). (2) *SGmodel* calculates the relative refractory ratio from scratch at each time bin. *SGfast* uses running approximations to the differential equations of which the exponentials in the relative refractory equation are solutions. This greatly reduces the computational load of each step of the simulation.

Calling syntax:
[spktimes, {nspikes}] = SGfast([dt, nrep], rate, {deadtime, refracparams})

where	spktimes	is a vector containing the times at which spikes occurred (sec).
	nspikes	is the number of spikes that occurred.
	dt	is the sample period of the rate function (sec). This is also used as the width of discrete time bins in the algorithm, and spike times will be multiples of *dt*.
	nrep	is the number of repetitions of the rate function.
	rate	is the rate function vector (spikes/sec).
	deadtime	is the dead time or absolute refractory period (sec). DEFAULT: 0.00075
	refracparams	is a vector [c0 s0 c1 s1] containing the parameters for the relative refractory period (dimensionless, sec, dimensionless, sec). DEFAULT: [0.5 0.001 0.5 0.0125]

Simulation of a Poisson Process without Refractoriness

SGfast makes use of the time-transformation method of simulating an inhomogeneous Poisson process, i.e. a Poisson process with a time-varying rate. Explanation of this method and mathematical references are given in Jackson and Carney (2005); here we give just a brief description of the procedure. Let $\lambda(t)$ be

the non-negative, time-varying rate for the Poisson process. (Note: If any of the rate values are negative, *SGfast* replaces those values with zeros.) Then the first step is to draw a sequence of independent and identically-distributed random variables $e_i, i = 0, 1, 2, \ldots$, from a unit-mean exponential distribution (using a pseudo-random number generator). Now let T_i be either the starting time (i.e. $i = 0$) of the entire process or the time of occurrence of the last spike (i.e. $i = 1, 2, 3, \ldots$) produced by the simulation. Then the time of occurrence of the next spike is the minimum time $T_{i+1} > T_i$ for which

$$\int_{T_i}^{T_{i+1}} \lambda(u)du \geq e_i.$$

This procedure is then iterated to determine all of the spike times necessary.

Adding Refractoriness

Let $0 \leq H(\tau) \leq 1$ be a refractory function, such that the time-varying rate of an inhomogeneous Poisson process with refractoriness is given by $R(t) = \lambda(t)\,[1 - H(t - T_i)]$, where T_i is the time of the last spike to occur prior to time t (see, e.g. Zhang et al., 2001, and Carney, 1993). Adding this type of refractory effect to the time-transformation procedure described above is simply accomplished by replacing the original rate function, $\lambda(t)$, with the new rate function $R(t)$. Thus, given that the last spike occurred at time T_i, the next spike will occur at the minimum time $T_{i+1} > T_i$ for which

$$\int_{T_i}^{T_{i+1}} \lambda(u)\,[1 - H(u - T_i)]\,du \geq e_i.$$

Once refractoriness is added to a Poisson process, the process is no longer Poisson, although it remains a renewal process. Thus, strictly speaking, to begin the simulation and calculate the first spike time, the integral above should not be started at time zero. Hence, *SGfast* draws a random interval from a homogeneous Poisson process with rate $\lambda(0)$ and assumes a spike occurred this much before time zero. Then the refractory effect at time zero of this spike is computed, and the simulation begins. In other words, if T_s is this interval drawn from a $\lambda(0)$-mean exponential distribution, then, at the beginning of the simulation, the first spike time T_1 is determined from the inequality

$$\int_{-T_s}^{0} \lambda(0)\,[1 - H(u + T_s)]\,du + \int_{0}^{T_1} \lambda(u)\,[1 - H(u + T_s)]\,du \geq e_0.$$

Faster Computation of the Refractory Function

The refractory function used in *SGmodel*, Zhang et al. (2001), and Carney (1993) is an absolute refractory period followed by a double exponential relative refractory period. This function is

$$H(\tau) = \begin{cases} 1.0, & \text{if } \tau < R_A; \\ c_0 e^{-(\tau - R_A)/s_0} + c_1 e^{-(\tau - R_A)/s_1}, & \text{if } \tau \geq R_A, \end{cases}$$

where R_A is the duration of the absolute refractory period and c_0, c_1, s_0, and s_1 are parameters of the relative refractory period. Obviously, when $t - T_i < R_A$, $H(t - T_i) = 1$, and the rate of the process $R(t - T_i)$ is zero. Thus, we can ignore, or "skip over", the interval $(T_i, T_i + R_A]$ after each spike time. But, the straightforward way to incorporate the relative refractory portion of the function into the simulation is to calculate $H(t - T_i)$, where T_i is the last spike time before time t, at each time step $t \geq T_i + R_A$ in the simulation. This is how it was accomplished in *SGmodel* and the aforementioned studies. However, this is computationally expensive. At each of these time steps in the simulation, four subtractions, two divisions, two exponentiations, and two multiplications must be computed, plus adding the two exponential functions together. The number of computations can be greatly reduced by using running approximations to the differential equations of which the exponentials are solutions. For instance, the exponential function $H_0(\tau) = c_0 e^{-(\tau - R_A)/s_0}$ is the solution of the differential equation

$$\frac{d}{d\tau} H_0(\tau) = -\frac{1}{s_0} H_0(\tau), \text{ for } \tau > R_A \text{ and } H_0(R_A) = c_0.$$

This differential equation can be approximated in discrete time by the difference equation

$$\begin{aligned} H_0(\tau + \Delta\tau) - H_0(\tau) &= -\frac{1}{s_0} H_0(\tau)\Delta\tau \\ H_0(\tau + \Delta\tau) &= H_0(\tau) - \frac{\Delta\tau}{s_0} H_0(\tau) \\ H_0(\tau + \Delta\tau) &= \left(1 - \frac{\Delta\tau}{s_0}\right) H_0(\tau). \end{aligned}$$

Therefore, during the simulation, at the end of an absolute refractory period, a variable, say H_0, is set equal to c_0. Then on each successive time step, H_0 is multiplied by the pre-calculated constant $K = 1 - \frac{\Delta\tau}{s_0}$ to yield the (approximate) value of the exponential at that time step. The same is done for the other exponential in the refractory function. Thus, at each time step, only two multiplications need to be computed, plus adding the values of the two exponentials together, in order to calculate the value of $H(\tau)$.

References

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